Counting problem: Seating arrangements.

Say 5 men, 5 women sitting in a row of 10 chairs.

Say all the seating arrangements are equally likely.

Let X denote the number of couples who are sitting together.

Find $E(X)$. Notice I do not ask for the mass of X, indeed the mass of X is challenging to compute!

Notice $X = X_1 + X_2 + \cdots + X_9$ where

X_j indicates if the jth pair of chairs has a couple in it.

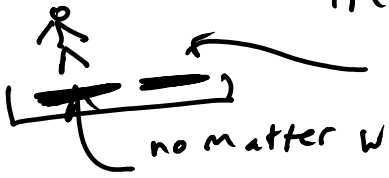
$X_j = 1$ if jth pair has a couple

$= 0$ otherwise.

$E(X) = E(X_1 + \cdots + X_9) = E(X_1) + \cdots + E(X_9)$

all nine of these are the same expectation.

Find $E(X_j) = \underset{\text{some } j.}{1}P(X_j = 1) + 0P(X_j = 0) = P(X_j = 1) = \frac{1}{9}$



there are 9 people equally likely to sit here, and exactly 1 is the partner of the person on the left

no matter who sits here (on the left)

$E(X) = \frac{1}{9} + \frac{1}{9} + \cdots + \frac{1}{9} = (9)(\frac{1}{9}) = 1$

So we expect exactly one couple to be sitting together.