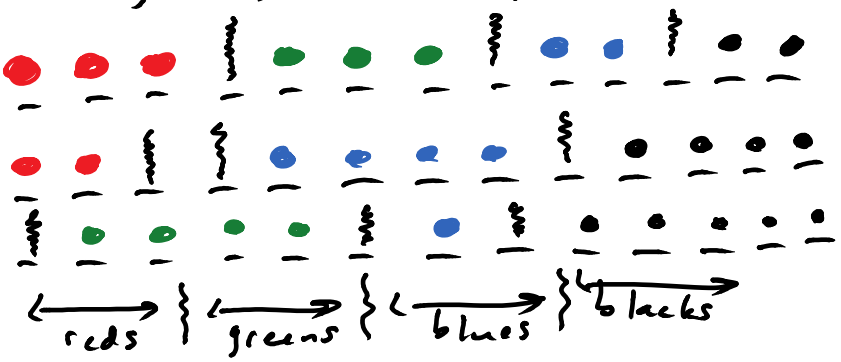
Counting: Say we want to pick 10 items from 4 types, with replacement, but without worrying about the order of selection. So at the end, we are OK to just group them by type.

E.g. Choose 10 items of types red, green, blue, black. We don't care about the order of selection, so just tells us how many many of each type



13 objects : 10 items and 3 = 4-1 dividing lines

locations of the bars give the full story, i.e. tell us how many of each color we have

$10 + 4 - 1 = 13$ objects

$4 - 1 = 3$ bars 10 balls

$\binom{13}{3} = \binom{13}{10}$ ways

In general, say we want r items (above $r = 10$) of n different types (above $n = 4$) and we do not care about the order of selection.

just want to know (x_1, x_2, x_3, x_4) in general $(x_1, x_2, \ldots, x_n)$

x_1 reds, x_2 greens, x_3 blues, x_4 blacks

So that $x_1 + x_2 + x_3 + x_4 = r$ } $x_1 + \ldots + x_n = r$

there are $r + n - 1$ objects total (balls + bars)

$n - 1$ bars r balls

$$\binom{r+n-1}{n-1} = \binom{r+n-1}{r}$$ ways to get r items of n types without regard to the order of selection.