Discrete Uniform Random Variable

Say X is a discrete uniform random variable if X takes on values $1, 2, 3, \ldots, n$, and all of these values are equally likely.

E.g. $p_X(x) = P(X = x) = \frac{1}{n}$ for $x = 1, 2, \ldots, n$

$E(X) = \frac{n+1}{2}$ Why? $E(X) = (1)(\frac{1}{n}) + (2)(\frac{1}{n}) + \cdots + (n)(\frac{1}{n})$

$= (\frac{1}{n})(1 + 2 + 3 + \cdots + n)$

$= (\frac{1}{n}) \frac{n(n+1)}{2}$

$= \frac{n+1}{2}$.

$Var(X) = \frac{n^2 - 1}{12}$.

Key example: X is the result on the roll of a 6-sided die (or any sided would work)

If X is discrete uniform on $1, 2, \ldots, 6$

then $E(X) = \frac{6+1}{2} = \frac{7}{2}$ $= 3.5$ $\quad Var(X) = \frac{6^2 - 1}{12} = \frac{35}{12}$ $= 2.9167$

We will also later have a continuous uniform random variable, and we will have much more to say about those.