Return to example where 20 CD's, 5 desirable, 15 undesirable, pick 3 and let X denote the number of desirable items among the 3.

$N = 20$
$M = 5, \quad N - M = 15$
$n = 3$

where $X_j = 1$ if jth item is desirable
$X_j = 0$ otherwise

$$E(X^2) = E((X_1 + X_2 + X_3)(X_1 + X_2 + X_3))$$
$$= E(X_1X_1) + E(X_1X_2) + E(X_1X_3)$$
$$+ E(X_2X_1) + E(X_2X_2) + E(X_2X_3)$$
$$+ E(X_3X_1) + E(X_3X_2) + E(X_3X_3)$$

all $E(X_iX_i)$ terms are the same

all $E(X_iX_j)$ terms are the same

Why? Just think about changing order of selection.

$$E(X^2) = 3E(X_1X_1) + \underbrace{(3)(2)}_{=6}E(X_1X_2)$$

here X_1 is an indicator random variable so $X_1X_1 = X_1$ $\left(\begin{array}{l}\text{because} \\ 1 \cdot 1 = 1 \\ 0 \cdot 0 = 0\end{array}\right)$

So $E(X_1X_1) = E(X_1) = \frac{5}{20} = \frac{1}{4}$

also X_1 and X_2 are indicators that are dependent on each other

$X_1 \cdot X_2 = 1$ if $X_1 = 1$ and $X_2 = 1$

$X_1 \cdot X_2 = 0$ otherwise

$$E(X_1X_2) = 1P(X_1X_2 = 1) + \cancel{0P(X_1X_2 = 0)}$$
$$= P(X_1 = 1 \text{ and } X_2 = 1)$$
$$= \underbrace{P(X_1 = 1)}_{\frac{5}{20}}\underbrace{P(X_2 = 1 \mid X_1 = 1)}_{\frac{4}{19}} = \frac{1}{4} \cdot \frac{4}{19} = \frac{1}{19}$$

$$E(X^2) = (3)\left(\frac{1}{4}\right) + (3)(2)\left(\frac{1}{19}\right)$$
$$= \frac{3}{4} + \frac{6}{19}$$