General technique (not specific to hypergeometrics)
for finding $E(X^2)$ when X is a sum of indicator random variables
(or any kind of random variables, summed).

Idea: Suppose $X = X_1 + X_2 + \cdots + X_n$.

Find
$$
\begin{aligned}
E(X^2) &= E((X_1+X_2+\cdots+X_n)(X_1+X_2+\cdots+X_n)) \\
&= E(X_1X_1 + X_1X_2 + X_1X_3 + \cdots + X_1X_n \\
&\quad + X_2X_1 + X_2X_2 + X_2X_3 + \cdots + X_2X_n \\
&\quad \cdots \\
&\quad + X_nX_1 + X_nX_2 + X_nX_3 + \cdots + X_nX_n) \\
&= E(X_1X_1) + E(X_1X_2) + \cdots \\
&\quad \cdots + E(X_nX_n). \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} E(X_iX_j)
\end{aligned}
$$

Question:
Can we do better, with regard to organizing. This is usually helpful:

Say $n=4$

$$
\begin{array}{llll}
E(X_1X_1) & +E(X_1X_2) & +E(X_1X_3) & +E(X_1X_4) \\
E(X_2X_1) & +E(X_2X_2) & +E(X_2X_3) & +E(X_2X_4) \\
E(X_3X_1) & +E(X_3X_2) & +E(X_3X_3) & +E(X_3X_4) \\
E(X_4X_1) & +E(X_4X_2) & +E(X_4X_3) & +E(X_4X_4)
\end{array}
$$

have two copies of all the off-diagonal terms.

$$\sum_{i=1}^{4} E(X_iX_i) + 2\sum_{1\le i<j\le 4} E(X_iX_j)$$

Same thing works in general: Nothing special for $n=4$.

$$E((X_1+\cdots+X_n)(X_1+\cdots+X_n)) = \sum_{i=1}^{n} E(X_iX_i) + 2\sum_{1\le i<j\le n} E(X_iX_j)$$

and all $E(X_iX_j)$ terms are the same.

In the special case where all $E(X_iX_i)$ terms are the same, this becomes:

$$nE(X_1X_1) + (n)(n-1)E(X_1X_2)$$

e.g. if $n=4$

$$4E(X_1X_1) + (4)(3)E(X_1X_2)$$

n

n^2-n
on diagonal
↓
$n(n-1)$
off diagonal