Example: Hypergeometric random variables

Say you and your roommate have 20 CD's, pick 3 to take with you, e.g. to a party. You really like 5 of the 20 CD's.
Within the 3 that you pick, let X denote the number of them that you really like. Find expected value and mass of X.

Here: 20 items altogether, so $N = 20$

5 of the 20 items are desirable so $M = 5$

automatically the other 15 items are undesirable, so $N - M = 15$

we pick 3 items, so $n = 3$.

$$E(X) = n\frac{M}{N} = (3)\frac{5}{20} = \frac{15}{20} = 3/4.$$

What about the mass?

$$p_X(0) = \frac{\binom{5}{0}\binom{15}{3}}{\binom{20}{3}}$$ ← i.e. get 0 out 5 of the desirable items, get 3 out 15 of the undesirable items.

$$= \frac{455}{1140} = 40\% = 0.40\ldots$$

$$p_X(1) = \frac{\binom{5}{1}\binom{15}{2}}{\binom{20}{3}} = \frac{525}{1140} = 46\% = 0.46\ldots$$

$$p_X(2) = \frac{\binom{5}{2}\binom{15}{1}}{\binom{20}{3}} = \frac{150}{1140} = 13\% = 0.13\ldots$$

$$p_X(3) = \frac{\binom{5}{3}\binom{15}{0}}{\binom{20}{3}} = \frac{10}{1140} = 1\% = 0.01\ldots$$

Check: $E(X) = (0)\left(\frac{455}{1140}\right) + (1)\left(\frac{525}{1140}\right) + (2)\left(\frac{150}{1140}\right) + (3)\left(\frac{10}{1140}\right) = \frac{855}{1140} = 3/4$

One last note: Write $X = X_1 + X_2 + X_3$ where X_j indicates if the jth item picked is desirable, so

$X_j = 1$ with prob $\frac{5}{20} = 1/4$

$X_j = 0$ otherwise

$E(X_j) = 1/4$

$$E(X) = E(X_1 + X_2 + X_3)$$
$$= E(X_1) + E(X_2) + E(X_3)$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$
$$= 3/4$$