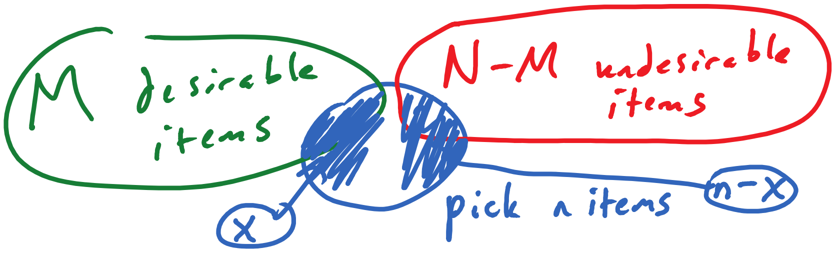
Hypergeometric random variables - Have 3 parameters

N total number of items present

M is the number of those items that are desirable
(the other $N-M$ items are undesirable)

n is the number of items we pick.

Say random variable X is Hypergeometric (N, M, n) if
X denotes the number of desirable items in a collection of
n items we pick. We pick without replacement,
and all choices are equally likely.



$X = x$ if x desirable items in our picked selection

Mass of X: $p_X(x) = P(X=x) = \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$

desirable items we need to pick

undesirable items we need

equally likely ways to pick the items

Expected value of X:

Think $X = X_1 + X_2 + \cdots + X_n$ where X_j indicates if the jth item picked is desirable.

The X_j's of course are dependent, e.g. if we get several desirable items at the start, this leaves fewer for later.

Even though they are dependent, OK, still decompose the expected value of the sum as the sum of the expected values:

$E(X) = E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$

Finally $E(X_j) = \frac{M}{N}$ for each j because ~~say~~ any of the N items could appear on the jth draw, and M of them are desirable. So altogether

$E(X) = \frac{M}{N} + \frac{M}{N} + \cdots + \frac{M}{N} = \frac{nM}{N}$.