Expected value and variance of Poisson random variables. We said that λ is the expected value of a Poisson(λ) random variable, but did not prove it. We did not (yet) say what the variance was. For the expected value, we calculate, for X that is a Poisson(λ) random variable:

$$\begin{aligned}
E(X) &= \sum_{x=0}^{\infty} x\frac{e^{-\lambda}\lambda^x}{x!} \\
&= \sum_{x=1}^{\infty} x\frac{e^{-\lambda}\lambda^x}{x!} && \text{since the } x=0 \text{ term is itself } 0 \\
&= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-1)!} && \text{divided on top and bottom by } x \\
&= \lambda e^{-\lambda}\sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} && \text{factor out } e^{-\lambda} \text{ and } \lambda \text{ too} \\
&= \lambda e^{-\lambda}\Big(\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\dots\Big) \\
&= \lambda e^{-\lambda}\sum_{x=0}^{\infty} \frac{\lambda^{x}}{x!} \\
&= \lambda e^{-\lambda}e^{\lambda} \\
&= \lambda
\end{aligned}$$

So in summary $E(X)=\lambda$. For $\mathrm{Var}(X)=E(X^2)-(E(X))^2=E((X)(X-1)+X)-(E(X))^2=E((X)(X-1))+E(X)-(E(X))^2=E((X)(X-1))+\lambda-\lambda^2$. Now we calculate

$$\begin{aligned}
E((X)(X-1)) &= \sum_{x=0}^{\infty} (x)(x-1)\frac{e^{-\lambda}\lambda^x}{x!} \\
&= \sum_{x=2}^{\infty} (x)(x-1)\frac{e^{-\lambda}\lambda^x}{x!} && \text{because } x=0 \text{ and } x=1 \text{ terms are themselves } 0 \\
&= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!} && \text{divide out by } x \text{ and } x-1 \\
&= \lambda^2 e^{-\lambda}\sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} && \text{factor out } e^{-\lambda} \text{ and } \lambda^2 \\
&= \lambda^2 e^{-\lambda}\Big(\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\dots\Big) && \text{(I had extra } e^{-\lambda} \text{ in the video on this line)} \\
&= \lambda^2 e^{-\lambda}e^{\lambda} \\
&= \lambda^2
\end{aligned}$$

In summary, $\mathrm{Var}(X)=\lambda^2+\lambda-\lambda^2=\lambda$.

So both the expected value and the variance of X are equal to λ.