Examples of Poisson random variables with inequalities, and with conditional probabilities. For instance, suppose that X is a Poisson($\lambda = 2.5$), as in the previous example.

Find $P(X \leq 4)$.

$$P(X \leq 4) = P(X = 0) + P(X = 1) + \ldots + P(X = 4) = 0.0821 + \ldots + 0.1336 = 0.8912.$$

What about $P(X = 2 \mid X \leq 4)$?

$$\begin{aligned} P(X = 2 \mid X \leq 4) &= \frac{P(X = 2 \ \& \ X \leq 4)}{P(X \leq 4)} \\ &= \frac{P(X = 2)}{P(X \leq 4)} \\ &= \frac{0.2565}{0.8912} \\ &= 0.2878. \end{aligned}$$

So, in other words, given that $X \leq 4$, the probability that $X = 2$ is 0.2878.

Another conditional probability example:

$$\begin{aligned} P(X > 3 \mid X > 1) &= \frac{P(X > 3 \ \& \ X > 1)}{P(X > 1)} \\ &= \frac{P(X > 3)}{P(X > 1)} \\ &= \frac{P(X = 4) + P(X = 5) + P(X = 6) + \ldots}{P(X = 2) + P(X = 3) + P(X = 4) + \ldots} \qquad \text{trouble!} \\ &\qquad \text{we do not have an easy way to sum these values, so instead...} \\ &= \frac{P(X > 3)}{P(X > 1)} \\ &= \frac{1 - P(X \leq 3)}{1 - P(X \leq 1)} \\ &= \frac{1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)}{1 - P(X = 0) - P(X = 1)} \\ &= \frac{1 - 0.0821 - 0.2052 - 0.2565 - 0.2138}{1 - 0.0821 - 0.2052} \\ &= \frac{0.2424}{0.7127} \\ &= 0.3401 \end{aligned}$$

I.e., given that $X > 1$, the probability that $X > 3$ is 0.3401.