Poisson random variables

These random variables apply when there is some kind of process, for which the average rate (say, per time period, for instance) is known. Need the occurrences in non-overlapping time periods to be independent.

What is the mass of a Poisson random variable X? Say that X has parameter λ. Then the mass is

$$p_X(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad \text{this is for integers } x \geq 0.$$

Why should this strange-looking mass even be a mass at all? First of all, all of the terms are positive. So it is enough to show that the terms add up to 1. Check:

$$\sum_{x=0}^{\infty} p_X(x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda}e^{\lambda} = 1.$$

So this is really a valid probability mass function.