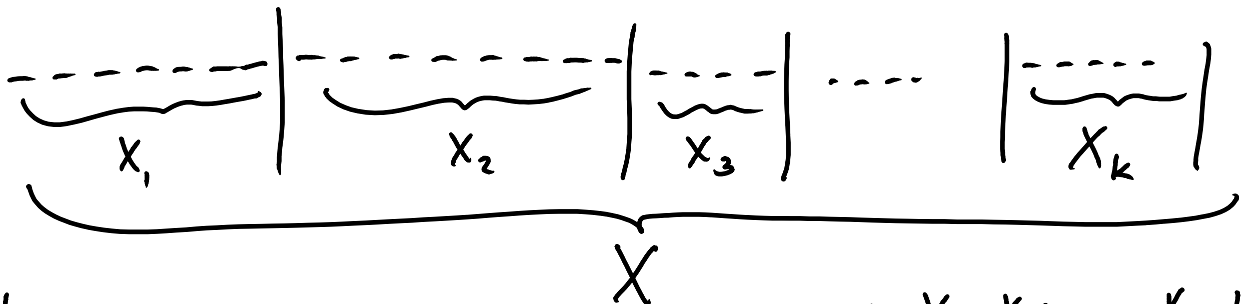
Sums of independent Negative Binomial random variables.

Say X_1 is a Negative Binomial (r_1, p)
X_2 is a Negative Binomial (r_2, p)
$\vdots$
X_k is a Negative Binomial (r_k, p)

must be the same p's.
X_i's must be independent.

Then $X = X_1 + X_2 + \cdots + X_k$, this makes X be Negative Binomial as well, with parameters $r = r_1 + r_2 + \cdots + r_k$ and parameter p.

Why??



how many successes occur altogether in the $X_1 + X_2 + \cdots + X_k$ trials? Exactly $r_1 + r_2 + \cdots + r_k$ of them. The trials are independent, each with probability of success p, so this automatically makes X have a Negative Binomial $(r = r_1 + \cdots + r_k, p)$ distribution.

E.g. Say X is Negative Binomial $(5, 1/3)$
Y is Negative Binomial $(9, 1/3)$
Z is Negative Binomial $(17, 1/3)$
and suppose X, Y, Z are independent.

Now define $U = X + Y + Z$. Then U is a Negative Binomial random variable too, with parameters $r = 5 + 9 + 17 = 31$
$p = 1/3$.

We do not even have to make a computation with the mass. We just get this property from the structure of U, i.e. from the way we built U as a sum of X, Y, Z.