Expected value and variance of Binomial random variables

Perhaps the easiest way to compute the expected value of a Binomial random variable is to use the interpretation that a Binomial(n, p) random variable is the sum of n independent Bernoulli(p) random variables. E.g., if X is a Binomial(n, p) random variable, we can think of

$$X = X_1 + X_2 + \ldots + X_n$$

where the X_i's are independent Bernoulli(p) random variables. Each X_i has expected value p, so

$$E(X) = E(X_1 + \cdots + X_n) = E(X_1) + \ldots + E(X_n) = p + \ldots + p = np.$$

For the variance, because the X_i's are independent, the variance of the sum is equal to the sum of the variance. So

$$\mathrm{Var}(X) = \mathrm{Var}(X_1 + \cdots + X_n) = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_n) = pq + \ldots pq = npq.$$