

Binomial coefficients :  $\binom{n}{j}$  read as "n choose j"  
 defined as  $\frac{n!}{j!(n-j)!}$

This is # of ways to pick  
 exactly j out n items in a row, without regard to the order  
 of picking  
 i.e. without noting order of selection.

$\binom{5}{3}$



5 possibilities

← 5 items, pick 3

with the order of selection noted,  
 there are  $(5)(4)(3)$  ways to pick.



4 remaining possibilities



3 remaining possibilities

$$(5)(4)(3) = \frac{5!}{2!} = 60$$

also there are  $3!$  ways this triple  
 could have been picked



2nd, 3rd, 5th  
items

so I overcounted by a factor of  $3!$   
 if I want to ignore the order of  
 selection.

So there are really only  
 $\frac{5!}{3!2!} = 10$  ways if we ignore  
 order of selection

10 ways: 1, 2, 3

1, 2, 4

without regarding

1, 2, 5

the order of selection

1, 3, 4

e.g. without coloring them as you pick them.

1, 3, 5

1, 4, 5

2, 3, 4

2, 3, 5

2, 4, 5

3, 4, 5

So if we have 5 trials, and we want  
 exactly 3 successes, there are

$$\binom{5}{3} = \frac{5!}{3!2!} = 10 \text{ ways that this could happen.}$$

binomial coefficients play a key role in defining Binomial random variables.