Define the <u>variance</u> of a random variable.

Let $\mu_X = E(X)$ for shorthand. This is a constant.

Variance of X is defined as $\underline{E((X-\mu_X)^2)} = Var(X)$.

Some useful properties of the variance:

1. Variance is a one-number way of describing the spread of a random variable. I.e. the amount to which the mass of the random variable is spread out around the expected value.
   E.g. 2 random variables can have the same expected value but really might be far spread around the expected value and the other is very well concentrated around the expected value.

2. $Var(X) \geq 0$ always. Can never be negative. Why?

$$Var(X) = E((X-\mu_X)^2) = E(h(X)) \quad \text{where } h(X) = (X-\mu_X)^2 \geq 0$$
$$= \sum_{\omega \in S} \underbrace{((X(\omega)-\mu_X)^2)}_{\geq 0} \underbrace{P(\{\omega\})}_{\geq 0} \geq 0$$

3. In practice, often use $Var(X) = E(X^2) - (E(X))^2$. Why??

$$Var(X) = E((X-\mu_X)^2) = E(X^2 - 2X\mu_X + \mu_X^2)$$
$$= E(X^2) - 2\mu_X \underbrace{E(X)}_{=\mu_X} + \mu_X^2$$
$$= E(X^2) - \mu_X^2$$
$$= E(X^2) - (E(X))^2$$

Lastly, we define the <u>standard deviation</u> of a random variable. Also measures spread around the expected value, just as the variance does.

<u>Std. dev. of X</u> is $\underline{\sigma_X = \sqrt{Var(X)}}$.

We will see that the standard deviation comes in the same "units" e.g. ft, seconds, etc as the random variable itself, when we get to continuous random variables, later in our study.