Define: Expected value of a function of a random variable

Say X is a random variable, say h is a function from $\mathbb{R}$ to $\mathbb{R}$

$$E[h(X)] = \sum_{\omega \in S} h(X(\omega)) P(\{\omega\})$$

another notation for this, if $\omega_1, \omega_2, \ldots$ are the possible outcomes, and if $X(\omega_j) = x_j$ i.e. x_j is the value X gets when ω_j is the outcome, then

$$E[h(X)] = \sum_j h(x_j) P(\{\omega_j\})$$

Another strategy is to group together all of the ω_j's that give a common value of X.

$$E[h(X)] = \sum_j h(x_j) \underbrace{P(X = x_j)}_{\text{mass of } X \text{ evaluated at } x_j}$$

$$E[h(X)] = \sum_j h(x_j) \, p_X(x_j)$$

Keep in mind that $h(X)$ is just a random variable too,

e.g. $S \xrightarrow{X} \mathbb{R} \xrightarrow{h} \mathbb{R}$

$\omega \longrightarrow X(\omega) \longrightarrow h(X(\omega))$