Example: Take a 6-sided die and roll it as many times as necessary, to see a certain value appear for the first time, e.g., roll the die until the first "3" appears. Let X denote the number of rolls that are needed. Find $E(X)$.

Idea: Define $X_j = 1$ if j or more rolls are needed, to see the first "3"
$\phantom{X_j = {}}0$ otherwise.

In other words, X_j is an indicator random variable for the event that j or more rolls are needed.

Claim $X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + \cdots\cdots$

Why? E.g. Say first "3" occurs on roll 5.

$X_1 = 1$ since ≥ 1 roll needed
$X_2 = 1$ since ≥ 2 rolls needed
$X_3 = 1$ since ≥ 3 rolls needed
$X_4 = 1$ since ≥ 4 rolls needed
$X_5 = 1$ since ≥ 5 rolls needed
$X_6 = 0$ since ≥ 6 rolls NOT needed
$X_7 = 0$ since ≥ 7 rolls NOT needed
.....

$X = 1 + 1 + 1 + 1 + 1 + 0 + 0 + 0 + 0 + \cdots\cdots$
$= 5$ in this example.

In general in this problem, $X = X_1 + X_2 + X_3 + \cdots\cdots$

$$E(X) = E(X_1 + X_2 + X_3 + \cdots\cdots)$$
$$= E(X_1) + E(X_2) + E(X_3) + \cdots\cdots$$
$$= 1 + 5/6 + (5/6)^2 + (5/6)^3 + \cdots\cdots =$$

$E(X_j) = P(A_j)$ where A_j indicates if j or more rolls needed. happens if and only if $j-1$ rolls not successful.

$= (5/6)^{j-1}$ So

$$E(X) = \left(1 + \tfrac{5}{6} + (\tfrac{5}{6})^2 + (\tfrac{5}{6})^3 + \cdots\right)\frac{(1-\tfrac{5}{6})}{(1-5/6)}$$
$$= \frac{1}{1/6}$$
$$= \boxed{6} \text{ just as before!}$$

Notice: no calculus used
no quotient rule
no derivatives, etc.!
simple!