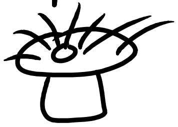
Example: consider n items, of which exactly 1 is "special" and the other $n-1$ items are not special. Let X = # of draws until the special item is found, when we draw the items blindly and without replacement, e.g. out of a hat.



E.g. $n=8$ 4th 1st 3rd 7th 2nd 6th 5th

i.e. special one "green" numbered the other $n-1$ balls, i.e. the nonspecial ones.

In this case, $X=5$.

Idea: Let A_j be the event that the jth labelled ball appears sometime before the special one. Here, for instance, A_4, A_1, A_3, A_7 occur but A_2, A_6, A_5 do not occur.

Let X_j indicate whether A_j happens i.e. $X_j = 1$ if A_j occurs, 0 otherwise.

$$X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + 1$$
$$= 1 + 0 + 1 + 1 + 0 + 0 + 1 + 1$$
$$= 5 \checkmark$$

This method works in general on this problem.

$$\text{So } E(X) = E(X_1 + X_2 + \cdots + X_{n-1} + 1)$$
$$= E(X_1) + E(X_2) + \cdots + E(X_{n-1}) + 1$$

Last observation:
$E(X_j) = \frac{1}{2}$ because each nonspecial ball has a 50/50 chance of appearing before the special ball.

$$E(X) = \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2} + \cdots + \tfrac{1}{2} + 1$$
$$= (n-1)(\tfrac{1}{2}) + 1$$
$$= \tfrac{n}{2} - \tfrac{1}{2} + 1$$
$$= \tfrac{n}{2} + \tfrac{1}{2}$$
$$= \frac{n+1}{2}$$

Notice: here we did not need to know that $1+2+\cdots+n = \frac{n(n+1)}{2}$.