Example: Let X denote the number of girls born in the births of 4 babies, i.e. how many are girls??

Idea: Define $X_j = \begin{cases} 1 & \text{if } j^{th} \text{ baby is a girl} \\ 0 & \text{otherwise} \end{cases}$

Notice: Always $X = X_1 + X_2 + X_3 + X_4$. It is just like counting on your fingers. E.g. if only babies 2 and 3 are girls, then

$X_1 = 0,\ X_2 = 1,\ X_3 = 1,\ X_4 = 0 \qquad X = 0+1+1+0 = 2$

Notice $E(X_j) = P(A_j)$ where A_j is the event that the j^{th} baby is a girl.

So $E(X_j) = \frac{1}{2}$ for each j.

So altogether $E(X) = E(X_1 + X_2 + X_3 + X_4)$

$= E(X_1) + E(X_2) + E(X_3) + E(X_4) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2.$

Same expected value we got as when we first treated the problem.

Also: notice we never computed the mass of X,

i.e. we never wrote down the numbers $\frac{1}{16}, \frac{4}{16}, \frac{6}{16}, \frac{4}{16}, \frac{1}{16}$.

So this method is easier, more straightforward, and more natural since we are used to counting occurrences (yes/no) on your fingers!