Joint mass of a collection of random variables, $X_1, X_2, \ldots, X_n$

$$p_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n)$$

Joint CDF of a collection of random variables $X_1, X_2, \ldots, X_n$

$$F_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = P(X_1 \le x_1, X_2 \le x_2, \ldots, X_n \le x_n)$$

Independence of the collection $X_1, \ldots, X_n$ is equivalent to either of these conditions (which are equivalent to each other)

$p_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = p_{X_1}(x_1) \cdots p_{X_n}(x_n)$ must hold for all $x_1, \ldots, x_n$

or equivalently

$F_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = F_{X_1}(x_1) \cdots F_{X_n}(x_n)$ again must hold for all $x_1, \ldots, x_n$.

Similarly if $A_1, \ldots, A_n$ are some events and if $X_1, \ldots, X_n$ are indicator random variables for those events i.e.

$X_j = 1 \iff A_j$ occurs

$X_j = 0$ otherwise

Then $X_1, \ldots, X_n$ independent if and only if the events $A_1, \ldots, A_n$ are independent.

Remember from studying events that $P(A \cap B) = P(A)P(B|A)$

Similarly we can write an equation for the joint mass of X and Y in terms of (say) the mass of X and the conditional mass of Y given X:

$$p_{X,Y}(x,y) = P(X = x, Y = y) = P(X = x)P(Y = y | X = x) = p_X(x)\,p_{Y|X}(y|x)$$

In summary: $p_{X,Y}(x,y) = p_X(x)\,p_{Y|X}(y|x)$.

Hint: When working towards independence of random variables, think about the analogous situation with events and try to do something similar. Often such a strategy will work.