Example of two random variables that are independent: Roll two dice, one of which is 4-sided, and the other is 6-sided. Let X be the value on the 4-sided die, and let Y be the value on the 6-sided die. If we assume that all 24 outcomes are equally likely, then X and Y are independent.

$$p_{X,Y}(x, y) = 1/24 = (1/4)(1/6) = p_X(x)p_Y(y)$$

works if $1 \leq x \leq 4$ and if $1 \leq y \leq 6$. Otherwise, the joint mass is 0.

Another view,

$$p_{X|Y}(x|y) = 1/4 = p_X(x)$$

for each $1 \leq x \leq 4$ and $1 \leq y \leq 6$.