Example of Bayes' Theorem (third version).

Suppose we put five different dice into a hat. The dice have the following number of sides: 4, 6, 8, 12, 20. When we choose a die from the hat, each of the five of the dice are equally likely to appear.

Suppose that a "3" appears. What is the probability it was the 4-sided die that was chosen?

Let A_1 be the event that the 4-sided die was chosen, A_2 be the event that the 6-sided die was chosen, A_3 be the event that the 8-sided die was chosen, A_4 be the event that the 12-sided die was chosen, and A_5 be the event that the 20-sided die was chosen. Notice that A_j's are disjoint (non-overlapping) and that the union of the A_j's is all of S. Let B denote the event that a "3" appears on the chosen die. Notice we don't know $P(B)$ either! We use this form of Bayes' Theorem:

$$P(A_j \mid B) = \frac{P(A_j)P(B|A_j)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3) + P(A_4)P(B|A_4) + P(A_5)P(B|A_5)}$$

In particular, we focus on $j = 1$ case.

$$P(A_1 \mid B) = \frac{(1/5)(1/4)}{(1/5)(1/4) + (1/5)(1/6) + (1/5)(1/8) + (1/5)(1/12) + (1/5)(1/20)} = 10/27 = 0.37$$

The probability that the 6-sided die was chosen, given that "3" appeared, is

$$P(A_2 \mid B) = \frac{(1/5)(1/6)}{(1/5)(1/4) + (1/5)(1/6) + (1/5)(1/8) + (1/5)(1/12) + (1/5)(1/20)} = 20/81 = 0.25$$

The probability that the 8-sided die was chosen, given that "3" appeared, is

$$P(A_3 \mid B) = \frac{(1/5)(1/8)}{(1/5)(1/4) + (1/5)(1/6) + (1/5)(1/8) + (1/5)(1/12) + (1/5)(1/20)} = 5/27 = 0.19$$

The probability that the 12-sided die was chosen, given that "3" appeared, is

$$P(A_4 \mid B) = \frac{(1/5)(1/12)}{(1/5)(1/4) + (1/5)(1/6) + (1/5)(1/8) + (1/5)(1/12) + (1/5)(1/20)} = 10/81 = 0.12$$

The probability that the 20-sided die was chosen, given that "3" appeared, is

$$P(A_5 \mid B) = \frac{(1/5)(1/20)}{(1/5)(1/4) + (1/5)(1/6) + (1/5)(1/8) + (1/5)(1/12) + (1/5)(1/20)} = 2/27 = 0.07$$

Finally, we notice that $P(A_1|B) + P(A_2|B) + P(A_3|B) + P(A_4|B) + P(A_5|B) = 1$, as we know it should.