Example of Bayes' Theorem with $B$ decomposed into $A \cap B$ or $A^c \cap B$. We know

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(A)P(B \mid A) + P(A^c)P(B \mid A^c)}$$

Suppose we have two dice in a hat (one has 6 sides, and one has 20 sides). Pick one of the dice at random (each die is chosen with probability 1/2). If we obtain a "5" on the die when we roll it, what is the probability that the die had 20 sides?

Let event $A$ denote the event that the 20-sided die was chosen. So $A^c$ is the event that the 6-sided die was chosen. Let event $B$ be the event that "5" appears on the chosen die when we roll it. (Notice we do not immediately have $P(B)$.)

$$P(A \mid B) = \frac{(1/2)(1/20)}{(1/2)(1/20) + (1/2)(1/6)} = 3/13 = .231$$

Know that $P(A) = 1/2$ and $P(A^c) = 1/2$ too. Note: $P(B \mid A) = 1/20$. Also note $P(B \mid A^c) = 1/6$.

In summary, if we get a "5" on the chosen die, it was the 20-sided die with probability $P(A \mid B) = 3/13 = .231$.

Automatically know the complementary probability (for free), i.e., the probability that the 6-sided die was chosen, given that a "5" appears, is $P(A^c \mid B) = 10/13 = .769$.