Conditional Probabilities are actually Probabilities

1. $0 \le P(A|B) \le 1$
2. $P(S|B) = 1$
3. If $A_j$'s disjoint then $P(\bigcup_j A_j | B) = \sum_j P(A_j|B)$

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Check: 1. $\frac{0}{P(B)} \le \frac{P(A \cap B)}{P(B)} \le \frac{P(B)}{P(B)}$, divide throughout by $P(B)$, get

$0 \le P(A|B) \le 1$

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2. $P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

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3. $P(\bigcup_j A_j | B) = \frac{P(\bigcup_j A_j \cap B)}{P(B)} = \frac{P(\bigcup_j (A_j \cap B))}{P(B)} = \sum_j \frac{P(A_j \cap B)}{P(B)} = \sum_j P(A_j|B)$