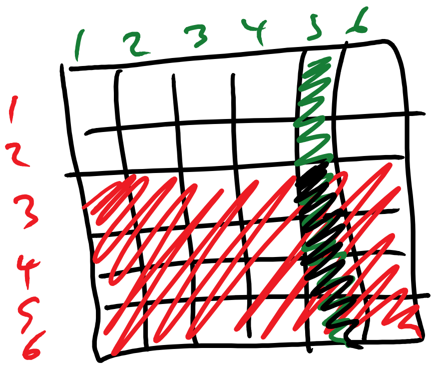
Q: Are independence and disjointness the same? No!



Event A be red die ≥ 3

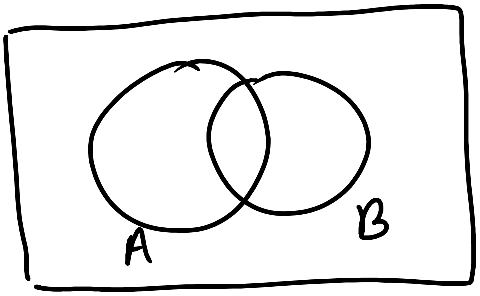
Event B be green die $= 5$

Calculated that A, B are independent.

Are A, B disjoint i.e. nonoverlapping? No.

4 outcomes are in both events

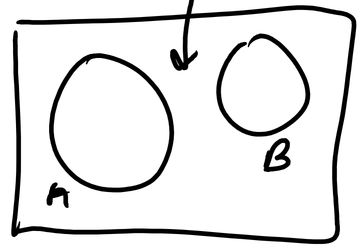
(3, 5), (4, 5), (5, 5), (6, 5)



A, B independent

A, B disjoint

no overlap



Can A, B be both independent and disjoint? Yes but only in a trivial kind of case. Only if $P(A) = 0$ or $P(B) = 0$.

$A \cap B = \phi$ since A, B disjoint

$P(A \cap B) = P(\phi) = 0$

$\quad \shortparallel$

$P(A)P(B)$ since A, B independent

$P(A)P(B) = 0$

This happens if and only if $P(A) = 0$ or $P(B) = 0$.

If $P(A) = 0$ then A is independent from all other events,

Why? $P(A \cap B) \leq P(A) = 0$ so $P(A \cap B) = 0$

since $A \cap B \subset A$

also $P(A) \cdot P(B) = 0$ (since $= 0$)

so $P(A \cap B) = 0 = P(A)P(B)$

So A, B independent.

If $A \subset B$ and $P(A) \neq 0$ and $P(B) \neq 1$ then A, B must be dependent.

$P(A \cap B) = P(A)$

because $A \cap B = A$

different probs.

$P(A)P(B) < P(A) = P(A \cap B)$

Since $P(B) < 1$

So A, B are dependent.