Equally likely outcomes, continued

If S has equally likely outcomes, and $S = \{x_1, \ldots, x_n\}$, and if A is an event, say $A = \{v_1, \ldots, v_k\}$, then what is the probability of A?

We can write

 $A = \{v_1\} \cup \{v_2\} \cup \cdots \cup \{v_k\}$

Notice that A is now written as a union of disjoint events, so

$$P(A) = P(\{v_1\}) + \dots + P(\{v_k\})$$

and since the outcomes were assumed to be equally likely at the start, we have $P(\{v_j\}) = 1/n$ for each j, so we get

$$P(A) = 1/n + 1/n + \dots + 1/n$$

or more simply

$$P(A)=k/n$$

If we write |A| = k, this means that, if all of the outcomes in S are equally likely, and |S| = n, then when event A has equally k of the outcomes, we have P(A) = k/n. Alternatively, P(A) = |A|/|S|.