The probability of the union of finitely many disjoint events is equal to the sum of the probabilities of the individual events. We did not assume this at the start. We only assumed that this works for infinite collections of events.

In other words, we want to show that if $A_1, \ldots, A_n$ are (pairwise) disjoint events, then

$$P\left(\bigcup_{j=1}^{n} A_j\right) = \sum_{j=1}^{n} P(A_j)$$

To do this, we define $A_{n+1} = \emptyset$ and $A_{n+2} = \emptyset$ and $A_{n+3} = \emptyset$ and so on. I.e., we let $A_j = \emptyset$ for all $j > n$. So

$$\bigcup_{j=1}^{n} A_j = \bigcup_{j=1}^{\infty} A_j$$

Thus the probabilities must be the same too

$$P\left(\bigcup_{j=1}^{n} A_j\right) = P\left(\bigcup_{j=1}^{\infty} A_j\right)$$

Since $A_1, A_2, \ldots$ are all (pairwise) disjoint, it follows from our basic assumption at the start (assumption/rule 3) that

$$P\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} P(A_j).$$

But $P(A_j) = P(\emptyset) = 0$ for $j > n$. So

$$\sum_{j=1}^{\infty} P(A_j) = \sum_{j=1}^{n} P(A_j).$$