STAT/MA 41600
In-Class Problem Set #39 part 2: November 14, 2018

1. As in question 1 on Problem Set 26, suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable X, and the time until the reply is denoted by Y. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of X and Y is

$$f_{X,Y}(x, y) = \frac{1}{750} e^{-(x/150+y/30)}$$

for $y > x > 0$, and $f_{X,Y}(x, y) = 0$ otherwise. Find the value of $\mathbb{E}(XY)$.

2. Same setup as the "Group Me" scenario in question 1.
2a. Find $\mathbb{E}(X)$.
2b. Find $\mathbb{E}(Y)$.
2c. Combining the answers to **1**, **2a**, **2b**, what is $\text{Cov}(X, Y)$?

3. (You are welcome to cite your work from previous problem sets, but please let the grader know which facts that you are citing, and where you got those facts.)

Draw five cards from a deck, *without replacement.* Let X denote the number of cards with pictures of people (Jacks, Queens, and Kings) that appear. Let $X_i = 1$ if the ith card has a picture of a person (Jack, Queen, King), and $X_i = 0$ otherwise.

Calculate the correlation $\rho(X_1, X_2)$ between the two variables X_1 and X_2.

4. (You are welcome to cite your work from previous problem sets, but please let the grader know which facts that you are citing, and where you got those facts.)

Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs. Let $X_i = 1$ if the ith bear pair is sitting together, and $X_i = 0$ otherwise.

Calculate the correlation $\rho(X_1, X_2)$ between the two variables X_1 and X_2.