STAT/MA 41600

In-Class Problem Set #37: November 9, 2018

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Problem Set 37 Answers

1a. The random variable X is a Binomial random variable with $n = 1000$ and $p = 2/6 = 1/3$.

1b. The exact probability is $P(330 < X < 340) = \sum_{x=331}^{339} \binom{1000}{x}(1/3)^x(2/3)^{1000-x}$.

1c. The approximation is $P(330 < X < 340) = P(330.5 < X < 339.5) = P\left(\frac{330.5-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}} < \frac{X-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}} < \frac{339.5-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}}\right) \approx P(-0.19 < Z < 0.41) = P(Z < 0.41) - P(Z \le -0.19) = P(Z < 0.41) - P(Z \ge 0.19) = P(Z < 0.41) - (1 - P(Z < 0.19)) = 0.6591 - (1 - 0.5753) = 0.2344.$

2a. The exact probability is $P(X = 325) = \binom{1000}{325}(1/3)^{325}(2/3)^{1000-325}$.

2b. The approximation is $P(X = 325) = P(324.5 < X < 325.5) = P\left(\frac{324.5-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}} < \frac{X-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}} < \frac{325.5-(1000)(1/3)}{\sqrt{(1000)(1/3)(2/3)}}\right) \approx P(-0.59 < Z < -0.53) = P(Z < -0.53) - P(Z \le -0.59) = P(Z \ge 0.53) - P(Z \ge 0.59) = (1 - P(Z < 0.53)) - (1 - P(Z < 0.59)) = (1 - 0.7019) - (1 - 0.7224) = 0.0205.$

3a. The random variable Y is a Gamma random variable with $r = 100{,}000$ and $\lambda = \frac{1}{0.86} = 1.1628$.

3b. The exact probability is $P(Y < 86{,}400) = \int_0^{86,400} \frac{(1/0.86)^{100,000}}{99,999!} x^{99,999} e^{-x/0.86}\, dx$.

3c. The approximate value is $P(Y < 86{,}400) = P\left(\frac{Y-(100,000)(0.86)}{\sqrt{(100,000)(0.86^2)}} < \frac{86,400-(100,000)(0.86)}{\sqrt{(100,000)(0.86^2)}}\right) \approx P(Z < 1.47) = 0.9292.$

4a. The random variable X is a Negative Binomial random variable with $n = 1000$ and $p = 0.62$.

4b. The exact probability is $P(X \le 1600) = \sum_{x=1000}^{1600} \binom{x-1}{999}(0.38)^{x-1000}(0.62)^{1000}$.

4c. The approximate value is $P(X \le 1600) = P(X \le 1600.5) = P\left(\frac{X-(1000)(1/0.62)}{\sqrt{(0.38)(1000)/(0.62^2)}} < \frac{1600.5-(1000)(1/0.62)}{\sqrt{(0.38)(1000)/(0.62^2)}}\right) \approx P(Z < -0.39) = P(Z > 0.39) = 1 - P(Z \le 0.39) = 1 - 0.6517 = 0.3483.$