STAT/MA 41600
In-Class Problem Set #37: November 9, 2018

1. Consider a group of students who pull out their beloved collection of small plastic toy bears, which they have not used since earlier in the semester. There are six different colors of bears. The students draw them repeatedly, with replacement after each draw. The students draw bears from buckets all day, making 1000 selections of bears altogether. Since they replace the bears after each selection, the colors of the selected bears are independent.

Let X denote the number of bears selected that are purple or orange.

1a. What kind of random variable is X? What are the parameter(s)?

1b. Write an expression for $P(330 < X < 340)$. (You do not need to find the exact value.)

1c. Use the Central Limit Theorem to approximate this probability.

2. Same setup as #1.

2a. Write an expression for $P(X = 325)$. (You do not need to find the exact value.)

2b. Use the Central Limit Theorem to approximate this probability.

3. Suppose we measure the time between snowflakes. We use X_1, X_2, X_3, ... to denote the times (in seconds) between consecutive snowflakes. Assume that these are independent exponential random variables, with expected value $\mathbb{E}(X_j) = 0.86$ for each j.

Let $Y = X_1 + \cdots + X_{100{,}000}$, i.e., let Y denote the time (in seconds) until the 100,000th snowflake arrives.

3a. What kind of random variable is Y? What are the parameter(s)?

3b. Write an expression for $P(Y < 86{,}400)$, i.e., for the probability that the time until the 100,000th snowflake is less than 86,400 seconds, i.e., less than 1 day. (You do not need to find the exact value.)

3c. Use the Central Limit Theorem to approximate this probability.

4. Suppose that we interview students as they enter Ross Ade Stadium. We continue to interview students until we find 1000 students who think Purdue will have an overall winning record this year. Suppose that the students' responses are independent. Also suppose that each student has probability 0.62 of saying "yes" (i.e., they think that Purdue will have an overall winning record), and probability 0.38 of saying "no."

Let X denote the total number of students needed, until we have found 1000 students who think that Purdue will have an overall winning record.

4a. What kind of random variable is X? What are the parameter(s)?

4b. Write an expression for $P(X \leq 1600)$. (You do not need to find the exact value.)

4c. Use the Central Limit Theorem to approximate this probability.