STAT/MA 41600
In-Class Problem Set #33: October 31, 2018
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Problem Set 33 Answers

1. We have

$$\begin{aligned}
P(X_1 + X_2 \le 30) &= \int_0^{30}\int_0^{30-x_1} (1/15)e^{-x_1/15}(1/15)e^{-x_2/15}\,dx_2\,dx_1 \\
&= \int_0^{30} (1/15)e^{-x_1/15}(-e^{-x_2/15})\big|_{x_2=0}^{30-x_1}\,dx_1 \\
&= \int_0^{30} (1/15)e^{-x_1/15}(1 - e^{-(30-x_1)/15})\,dx_1 \\
&= \int_0^{30} ((1/15)e^{-x_1/15} - (1/15)e^{-2})\,dx_1 \\
&= (1 - e^{-2} - 2e^{-2})\,dx_1 \\
&= 1 - 3e^{-2} \\
&= .5940
\end{aligned}$$

and therefore $P(X_1 + X_2 > 30) = 1 - P(X_1 + X_2 \le 30) = 3e^{-2} = 0.4060$.

2a. This is a Gamma random variable with parameters $r = 700$ and $\lambda = 1/5$.
2b. The expected time is $r/\lambda = (700)(5) = 3500$ seconds.
2c. The variance is $r/\lambda^2 = (700)(25) = 17500$.

3a. No, $X + Y$ is not a Gamma random variable, because they have different λ parameters.
3b. We compute

$$\begin{aligned}
P(X_1 + X_2 \le 9) &= \int_0^{9}\int_0^{9-x_1} (1/10)e^{-x_1/10}(1/3)e^{-x_2/3}\,dx_2\,dx_1 \\
&= \int_0^{9} (1/10)e^{-x_1/10}(-e^{-x_2/3})\big|_{x_2=0}^{9-x_1}\,dx_1 \\
&= \int_0^{9} (1/10)e^{-x_1/10}(1 - e^{-(9-x_1)/3})\,dx_1 \\
&= \int_0^{9} ((1/10)e^{-x_1/10} - (1/10)e^{7x_1/30-3})\,dx_1 \\
&= 1 + (3/7)e^{-3} - (10/7)e^{-9/10} \\
&= .4405
\end{aligned}$$

4a. The random variable V is an exponential random variable with parameter $\lambda = 1/75 + 1/35 + 1/50 = 13/210$.
4b. The CDF of V is $F_V(a) = 1 - e^{-(13/210)(a)}$ for $a > 0$. So the median value "a" of V satisfies $1 - e^{-(13/210)(a)} = 1/2$, so $1/2 = e^{-(13/210)(a)}$, so $\ln(1/2) = -(13/210)(a)$, so the median is $a = -(210/13)\ln(1/2) = 11.1970$.