STAT/MA 41600
In-Class Problem Set #32 part 2: October 29, 2018
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Problem Set 32 part 2 Answers

1. We have $P(X > U) = \int_0^a\int_u^\infty (1/a)\lambda e^{-\lambda x}\,dx\,du = \int_0^a (1/a)(-e^{-\lambda x})|_{x=u}^{\infty}\,du = \int_0^a (1/a)(e^{-\lambda u})\,du = (-1/a)(1/\lambda)(e^{-\lambda u})|_{u=0}^{a} = (1/a)(1/\lambda)(1-e^{-\lambda a})$.

2. We compute

$$\begin{aligned}
P(X_1 + X_2 \le 25) &= \int_0^{25}\int_0^{25-x_1} (1/10)e^{-x_1/10}(1/10)e^{-x_2/10}\,dx_2\,dx_1 \\
&= \int_0^{25} (1/10)e^{-x_1/10}(-e^{-x_2/10})|_{x_2=0}^{25-x_1}\,dx_1 \\
&= \int_0^{25} (1/10)e^{-x_1/10}(1-e^{-(25-x_1)/10})\,dx_1 \\
&= \int_0^{25} ((1/10)e^{-x_1/10} - (1/10)e^{-5/2})\,dx_1 \\
&= 1 - (7/2)e^{-5/2} \\
&= 0.7127
\end{aligned}$$

3. We let X, Y, Z denote the times (in seconds) until the phone rings, email arrives, or computer beeps. Then $P(X \ge 10 \;\&\; Y \ge 10 \;\&\; Z \ge 10) = P(X \ge 10)P(Y \ge 10)P(Z \ge 10) = e^{-10/30}e^{-10/20}e^{-10/15} = e^{-3/2} = 0.2231$.

Alternatively, we can let W denote the minimum of X, Y, Z. Since the minimum of independent exponential random variables is also an exponential random variable, and we add the parameters, then W is an exponential random variable with parameter $\lambda = 1/30 + 1/20 + 1/15 = 3/20$. So we get $P(X \ge 10 \;\&\; Y \ge 10 \;\&\; Z \ge 10) = P(W \ge 10) = e^{-(3/20)(10)} = e^{-3/2}$.

4. We compute

$$\begin{aligned}
P(|X-Y| \le 1) &= P(X \le Y \le X+1) + P(Y \le X \le Y+1) \\
&= \int_0^\infty\int_x^{x+1} (1/3)e^{-x/3}(1/4)e^{-y/4}\,dy\,dx + \int_0^\infty\int_y^{y+1} (1/3)e^{-x/3}(1/4)e^{-y/4}\,dx\,dy \\
&= \int_0^\infty (1/3)e^{-x/3}(-e^{-y/4})|_{y=x}^{x+1}\,dx + \int_0^\infty (-e^{-x/3})|_{x=y}^{y+1}(1/4)e^{-y/4}\,dy \\
&= \int_0^\infty (1/3)e^{-x/3}(e^{-x/4} - e^{-(x+1)/4})\,dx + \int_0^\infty (e^{-y/3} - e^{-(y+1)/3})(1/4)e^{-y/4}\,dy \\
&= \int_0^\infty (1/3)(e^{-7x/12} - e^{-1/4}e^{-7x/12})\,dx + \int_0^\infty (e^{-7y/12} - e^{-1/3}e^{-7y/12})(1/4)\,dy \\
&= (1/3)((-12/7)e^{-7x/12} - (-12/7)e^{-1/4}e^{-7x/12})|_{x=0}^{\infty} \\
&\quad + ((-12/7)e^{-7y/12} - (-12/7)e^{-1/3}e^{-7y/12})(1/4)|_{y=0}^{\infty} \\
&= (4/7)(1-e^{-1/4}) + (3/7)(1-e^{-1/3}) \\
&= 0.2479
\end{aligned}$$