STAT/MA 41600
In-Class Problem Set #26: October 15, 2018

1. Suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable X, and the time until the reply is denoted by Y. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of X and Y is

$$f_{X,Y}(x,y) = \frac{1}{750} e^{-(x/150+y/30)}$$

for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.
1a. Are X and Y independent?
1b. Find $P(Y > 2X)$.

2. Suppose that, for some constant value k, the random variables X, Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} ke^{-(x/150+y/30)} & \text{for } x > 0 \text{ and } y > 0, \\ 0 & \text{otherwise.} \end{cases}$$

2a. What is the value of the constant k? (It is not the case that $k = 1/750$.)
2b. Are X and Y independent?
2c. What is the probability density function of X?
2d. What is the probability density function of Y?

3. Suppose that X and Y have a constant joint probability density function on the triangle with vertices at $(0,0)$, $(0,6)$, $(10,0)$, so that $f_{X,Y}(x,y) = 1/30$ for values (x,y) in this triangle, and $f_{X,Y}(x,y) = 0$ otherwise.
3a. Are X and Y independent?
3b. Find $P(5Y < 3X)$.

4. Suppose that, for some constant value k, the random variables X, Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} k(3-x)(4-y) & \text{for } 0 < x < 3 \text{ and } 0 < y < 4, \\ 0 & \text{otherwise.} \end{cases}$$

4a. What is the value of the constant k?
4b. Are X and Y independent?
4c. What is the probability density function of X?
4d. What is the probability density function of Y?