STAT/MA 41600
In-Class Problem Set #4: August 29, 2018
Solutions by Mark Daniel Ward

Problem Set 4 Answers

1. Let A be the event that she selected the dodecahedron, and let B the event that she rolled a 10. Then $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$. We note that $P(A \cap B) = (1/5)(1/12) = 1/60$ and $P(B) = (1/5)(0) + (1/5)(0) + (1/5)(0) + (1/5)(1/12) + (1/5)(1/20) = 2/75$. So we get $P(A \mid B) = (1/60)/(2/75) = 5/8 = 0.625$.

2a. If Alice receives a flush, this leaves 3 complete suits (of 13 cards each) and a 4th suit that has only 8 cards. The conditional probability that Bob receives a flush of any particular one of the first three types is $(13/47)(12/46)(11/45)(10/44)(9/43) = 39/46483$. The conditional probability that Bob receives a flush of the last type is $(8/47)(7/46)(6/45)(5/44)(4/43) = 56/1533939$. So the overall conditional probability that Bob receives a flush, given that Alice received a flush, is $(3)(39/46483) + 56/1533939 = 3917/1533939 = 0.002554$.
2b. If Alice receives a full house, this leaves 2 complete suits (of 13 cards each), a 3rd suit that only has 11 cards, and a 4th suit that has only 10 cards.

The conditional probability that Bob receives a flush of any particular one of the first two types is $(13/47)(12/46)(11/45)(10/44)(9/43) = 39/46483$. The conditional probability that Bob receives a flush of the third type is $(11/47)(10/46)(9/45)(8/44)(7/43) = 14/46483$. The conditional probability that Bob receives a flush of the fourth type is $(10/47)(9/46)(8/45)(7/44)(6/43) = 84/511313$. So the overall conditional probability that Bob receives a flush, given that Alice received a full house, is $(2)(39/46483) + 14/46483 + 84/511313 = 1096/511313 = 0.002144$.

3. Let A denote the event that the all-blue die is selected. Let B denote the event that a blue side is selected. Then $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$. We have $P(A \cap B) = (1/5)(1)$ and $P(B) = (1/5)(1) + (1/5)(3/4) + (1/5)(2/4) + (1/5)(1/4) + (1/5)(0) = 1/2$. So we conclude that $P(A \mid B) = (1/5)/(1/2) = 2/5$.

4. Given that the first bear that is purple or orange appears on the 4th draw, then the first three draws are independent and are each equally likely to be any of the four remaining colors. So we get:
4a. The conditional probability that the first three draws have 0 blue bears is $\binom{3}{0}(1/4)^0(3/4)^3 = 27/64$.
4b. The conditional probability that the first three draws have 1 blue bear is $\binom{3}{1}(1/4)^1(3/4)^2 = 27/64$.
4c. The conditional probability that the first three draws have 2 blue bears is $\binom{3}{2}(1/4)^2(3/4)^1 = 9/64$.
4d. The conditional probability that the first three draws have 3 blue bears is $\binom{3}{3}(1/4)^3(3/4)^0 = 1/64$.