STAT/MA 41600
In-Class Problem Set #3: August 27, 2018
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Problem Set 3 Answers

1a. Method #1: We can treat the emails related to class as "good"; the emails related to research as "bad"; and the ones unrelated to work as "neutral"; so the probability is $(.30)/(.30 + .45) = 0.4$.

Method #2: If we let j denote the number of emails unrelated to work, the probability is $\sum_{j=1}^{\infty}(.25)^{j-1}(.30) = \frac{.30}{1-.25} = 0.4$.

1b. Method #1: It takes 7 or more emails to get one unrelated to work, if and only if the first six are related to work, which happens with probability $(.30 + .45)^6 = (.75)^6 = 0.1780$.

Method #2: We compute directly, using j to denote the number of emails needed, to get the first one that is unrelated to work. We have $\sum_{j=7}^{\infty}(.30 + .45)^{j-1}(.25) = \sum_{j=7}^{\infty}(.75)^{j-1}(.25) = (.75)^6 \sum_{j=1}^{\infty}(.75)^{j-1}(.25) = \frac{(.75)^6(.25)}{1-.75} = (.75)^6 = 0.1780$.

2a. Method #1: Let "good" denote different die values, with sum strictly more than 8; "bad" denote different die values, with sum less than or equal to 8; and "neutral" means the die values are the same. Then the probability is $\frac{8/36}{8/36+22/36} = 8/30 = 4/15$.

Method #2: We compute directly, using j to denote the number of rolls needed until the first one where their values are different. We have $\sum_{j=1}^{\infty}(6/36)^{j-1}(8/36) = \frac{8/36}{1-6/36} = 8/30 = 4/15$.

2b. Method #1: Let "good" denote different die values, with sum strictly less than 8; "bad" denote different die values, with sum greater than or equal to 8; and "neutral" means the die values are the same. Then the probability is $\frac{18/36}{18/36+12/36} = 18/30 = 3/5$.

Method #2: We compute directly, using j to denote the number of rolls needed until the first one where their values are different. We have $\sum_{j=1}^{\infty}(6/36)^{j-1}(18/36) = \frac{18/36}{1-6/36} = 18/30 = 3/5$.

2c. Method #1: Let "good" denote different die values, with sum exactly 8; "bad" denote different die values, with sum not equal to 8; and "neutral" means the die values are the same. Then the probability is $\frac{4/36}{4/36+26/36} = 4/30 = 2/15$.

Method #2: We compute directly, using j to denote the number of rolls needed until the first one where their values are different. We have $\sum_{j=1}^{\infty}(6/36)^{j-1}(4/36) = \frac{4/36}{1-6/36} = 4/30 = 2/15$.

3a. Restricting attention to only the R&B songs and the rap songs, we note that a song is R&B with probability $(.30)/(.30 + .20) = 3/5$, or a song is rap with probability $(.20)/(.30 + .20) = 2/5$. So the desired probability is $(2/5)(2/5)(3/5) = 0.096$.

3b. The probability is $(.20)/(.20 + .15 + .35) = 2/7$; here we are viewing R&B as "neutral", rap as "good", and the other genres (reggae and hip hop) as "bad".

4a. Method #1: It takes strictly more than 8 bears that are not purple or orange, to get one that is purple or orange, if and only if the first eight are not purple or orange, which happens with probability $(2/3)^8 = 0.0390$.

Method #2: We compute directly, using j to denote the number of bears needed, to get the first one that is purple or orange. We have $\sum_{j=9}^{\infty}(2/3)^{j-1}(1/3) = (2/3)^8 \sum_{j=1}^{\infty}(2/3)^{j-1}(1/3) = \frac{(2/3)^8(1/3)}{1-2/3} = (2/3)^8 = 0.0390$.

4b. The probability is $\sum_{j-1}^{7}(2/3)^{j-1}(1/3) = (1/3)(1 + 2/3 + \cdots + (2/3)^6) = (1/3)(1 + 2/3 + \cdots + (2/3)^6)\frac{1-2/3}{1-2/3} = 1 - (2/3)^7 = 0.9415$.

4c. The probability is $(2/3)^7(1/3) = 0.0195$.

Extra brain stretch: Yes, it is possible, but only with one method, namely, using one die labelled 1, 2, 2, 3, 3, 4, and the other die labelled 1, 3, 4, 5, 6, 8. These are called Sicherman dice.