$\begin{array}{c} {\rm STAT/MA~41600}\\ {\rm Midterm~Exam~\#1:~October~6,~2017} \end{array}$

Name _____

Purdue student ID (10 digits)

1. The testing booklet contains 5 questions, but students only need to answer 4 of the questions. The 4 questions chosen by the student will all be weighted evenly (i.e., each question is worth 1/4 of the midterm exam grade).

2. Permitted Texas Instruments calculators: BA-35 BA II Plus* BA II Plus Professional Edition* TI-30XS MultiView* TI-30Xa TI-30XIIS* TI-30XIIS* TI-30XIIB* TI-30XB MultiView* *The memory of the calculator should be cleared at the start of the exam.

3. Circle your final answer in your booklet; otherwise, no credit may be given.

4. There is no penalty for guessing or partial work.

5. Show all your work in the exam booklet. If the majority of questions are answered correctly, but insufficient work is given, the exam could be considered for academic misconduct.

6. Extra sheets of paper are available from the proctor.

1. Consider 3 students who wear similar-looking black coats to a party. When they leave the party, they are in a hurry, and they each randomly grab a coat as they depart, with all possibilities equally likely. Use inclusion-exclusion to calculate the probably that none of them get their correct coat. [Hint: First find the probability that *at least one* of them gets their correct coat.]

2. Suppose that X and Y are independent random variables, such that X has probability mass function $p_X(x) = (1/3)(2/3)^{x-1}$ for integers $x \ge 1$, and Y has probability mass function $p_Y(y) = (3/4)(1/4)^{y-1}$ for integers $y \ge 1$.

2a. What kind of variables are X and Y? What are their parameters?

2b. What is Var(X - Y)?

2c. Find the probability that X and Y are equal.

3. A bag of candy contains 10 green M&M's and 10 red M&M's. Suppose that 10 students pick 2 candies each, without replacement. Let X denote the number of students who get one red and one green candy.

3a. Find $\mathbb{E}(X)$.

3b. Find Var(X).

4. Suppose that X and Y are independent Hypergeometric random variables that each have parameters N = 6, M = 3, and n = 2.

4a. What is the probability mass function of X?

4b. What is the probability that X and Y are equal, i.e., what is P(X = Y)?

5. Roll five (6-sided) dice. Let D be the event that the sum of the values on the five dice is exactly 8.

Let A be the event that exactly 4 of the 5 dice have the value "1".

Let B be the event that exactly 3 of the 5 dice have the value "1".

Let C be the event that exactly 2 of the 5 dice have the value "1".

5a. Find $P(A \mid D)$.

5b. Find $P(B \mid D)$.

5c. Find $P(C \mid D)$.

[Hint: Given that event D occurs, we know that A or B or C occurs, and these three events are disjoint, so we should have $P(A \mid D) + P(B \mid D) + P(C \mid D) = 1$.]