STAT/MA 41600
In-Class Problem Set #44: December 8, 2017

1. Consider a circle of radius 3. Pick a point uniformly at random in this circle. In other words, the probability that the point is in a region within this circle is *proportional* to the area of that region.

1a. What is the probability that the point is in a circle of radius $1/5$, centered at the origin?

1b. What is the probability that the point is in a circle of radius $7/3$, centered at the origin?

1c. In general, for $0 \le a \le 3$, what is the probability that the point is in a circle of radius a, centered at the origin?

1d. Let X be the distance from the point to the center of the circle. What is the CDF of X? [Hint: See your solution to 1c.]

1e. What is the probability density function $f_X(x)$ of X?

1f. Check that the density you found in part e is valid, i.e., nonnegative and integrates to 1.

2a. Suppose that X is an exponential random variable with parameter $\lambda > 0$, i.e., with $\mathbb{E}(X) = 1/\lambda$. Define $Y = 5X$. What kind of random variable is Y? What is the parameter of Y? What is the expected value? (Hint: It might be easiest to consider the CDF of Y and compare it to the CDF of X.)

2b. Jorge waits for a bus. Suppose that his waiting time V, measured in minutes, is an exponential random variable, with a mean waiting time of 3 minutes.

Now let W denote his waiting time, measured in seconds. What kind of random variable is W? What is the parameter of W? What is the expected value?

3. Suppose that X is a Binomial random variable with parameters $n = 5$ and $p = 1/7$. Define $Y = 3X$.

3a. Is Y a Binomial random variable? Why or why not?

3b. What are the expected value and variance of Y?

4. Review question: Apple decides to start making the iPhone in a rainbow array of colors. Suppose a family gets excited about this promotion and buys iPhones in 7 different colors (1 per color). The family also buys 7 protective cases, in the same colors. They individually wrap these 14 gifts (7 iPhones and 7 protective cases).

Of course it is easy to distinguish—even when the gifts are wrapped—which ones are iPhones and which ones are protective cases, because the iPhone boxes are bigger.

On the night before they open the presents—horror of horrors!—they discover that their baby has played with the presents and randomly rearranged them (with all possible arrangements being equally likely). The parents are too tired to unwrap and rewrap the gifts. As a result, each of the 7 family members will get a randomly selected iPhone and a randomly selected protective case on the morning of the holiday.

Let X denote the number of family members who get a matching color iPhone and protective case. Find the expected value and the variance of X.

4a. Is X a Binomial random variable? Why or why not?

4b. What are the expected value and variance of X?

It has been a true pleasure to work with all of you this semester. Please stop by to see me if you would like to review the material, or for questions anytime.