STAT/MA 41600
In-Class Problem Set #40: November 27, 2017

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = 21e^{-3x-4y}$ for $0 < y < x$, and $f_{X,Y}(x,y) = 0$ otherwise.

Find $\mathbb{E}(X \mid Y = 2)$, i.e., the expected value of X, given $Y = 2$.

2. Suppose that, at the start of the day, a jar contains a random number of bears that has a Poisson distribution with mean 18. During the day, each such bear is randomly painted red, blue, or yellow, with each of these three possibilities being equally likely, and with the painting of bears being independent. Suppose that, at the end of the day, we discover that there are 7 red bears and 3 blue bears.

2a. Given all of the information above, what kind of random variable is the number of yellow bears at the end of the day?

2b. How many yellow bears do we expect?

3. Suppose X and Y have joint probability mass function $p_{X,Y}(x,y) = \frac{x}{7y}$ for $1 \leq x \leq y \leq 4$, and $p_{X,Y}(x,y) = 0$ otherwise.

Find $\mathbb{E}(X \mid Y = 3)$, i.e., the expected value of X, given $Y = 3$.

4. For some additional practice, we are wrapping two unrelated problems into question 4. (Questions 4a and question 4b really are separate problems!)

4a. Consider a pair of random variables X and Y whose joint probability density function is constant on the triangle with vertices at the points $(-4, 0)$, $(0, 2)$, and $(8, 0)$.

Find $\mathbb{E}(X \mid Y = 1)$, i.e., the expected value of X, given $Y = 1$.

4b. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = (3/4)(x-y)$ for $0 < y < x < 2$, and $f_{X,Y}(x,y) = 0$ otherwise.

Find $\mathbb{E}(X \mid Y = 1/3)$, i.e., the expected value of X, given $Y = 1/3$.