STAT/MA 41600
In-Class Problem Set #37: November 10, 2017
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Problem Set 37 Answers

Throughout this problem set, we use Z to denote a standard normal random variable, i.e., with mean 0 and variance 1.

1a. Let X denote the number of 3's, so X is Binomial with $n = 100$ and $p = 1/5$. So we get $P(X \geq 18) = P(X \geq 17.5) = P\Big(\frac{X-(100)(1/5)}{\sqrt{(100)(1/5)(4/5)}} \geq \frac{17.5-(100)(1/5)}{\sqrt{(100)(1/5)(4/5)}}\Big) \approx P(Z \geq -0.63) = P(Z \leq 0.63) = 0.7357$.
1b. Let Y denote the number of even values, so Y is Binomial with $n = 100$ and $p = 3/5$. So we get $P(Y \leq 55) = P(Y \leq 55.5) = P\Big(\frac{Y-(100)(3/5)}{\sqrt{(100)(3/5)(2/5)}} \leq \frac{55.5-(100)(3/5)}{\sqrt{(100)(3/5)(2/5)}}\Big) \approx P(Z \leq -0.92) = P(Z \geq 0.92) = 1 - P(Z \leq 0.92) = 1 - 0.8212 = 0.1788$.

2. Let X denote the number of chocolate chip cookies, so X is Binomial with $n = 112$ and $p = 0.4$. So we get $P(X \geq 50) = P(X \geq 49.5) = P\Big(\frac{X-(112)(0.4)}{\sqrt{(112)(0.4)(0.6)}} \geq \frac{49.5-(112)(0.4)}{\sqrt{(112)(0.4)(0.6)}}\Big) \approx P(Z \geq 0.91) = 1 - P(Z \leq 0.91) = 1 - 0.8186 = 0.1814$.

3. We have $P(U_1 + \cdots + U_{240} \geq 1000) = P\Big(\frac{U_1+\cdots+U_{240}-(240)(4)}{\sqrt{(240)(25/3)}} \geq \frac{1000-(240)(4)}{\sqrt{(240)(25/3)}}\Big) \approx P(Z \geq 0.89) = 1 - P(Z \leq 0.89) = 1 - 0.8133 = 0.1867$.

4a. The X_j's are each exponential random variables with parameter $\lambda = 2$.
4b. Yes, they are independent, because the joint probability density function can be factored as $f_{X_1,\ldots,X_{80}}(x_1, \ldots, x_{80}) = (2e^{-2x_1}) \cdots (2e^{-2x_{80}})$ when all of the x_j's are positive, and $f_{X_1,\ldots,X_{80}}(x_1, \ldots, x_{80}) = 0$ otherwise.
4c. Since X_j's are each exponential random variables with parameter $\lambda = 2$, then $\mathbb{E}(X_j) = 1/\lambda = 1/2$ and $\text{Var}(X_j) = 1/\lambda^2 = 1/4$.
4d. The random variable Y has a Gamma distribution with $r = 80$ and $\lambda = 2$.
4e. We have $P(Y < 45) = P(X_1 + \cdots + X_{80} < 45) = P\Big(\frac{X_1+\cdots+X_{80}-(80)(1/2)}{\sqrt{(80)(1/4)}} < \frac{45-(80)(1/2)}{\sqrt{(80)(1/4)}}\Big) \approx P(Z < 1.12) = 0.8686$.