STAT/MA 41600

In-Class Problem Set #37: November 10, 2017

1. As in question 4 on Problem Set 15, Alice rolls a die until the first occurrence of the value "1" and then she stops. Suppose that today it takes her exactly 101 rolls to accomplish this, i.e., she rolls 100 times without getting any roll of "1", and then the 101st roll is a "1".

Conditional on the information above:

1a. Approximate the probability that the value "3" appears 18 or more times altogether.

1b. Approximate the probability that an even value appears on at most 55 of the rolls.

2. As in question 2 on Problem Set 17, starting on the first Sunday of the semester, Carlos randomly grabs a cookie as he exits the dining court at lunch. Assume that 40% of the cookies are chocolate, and that his picks are independent from day to day.

He does this all semester. Assume that, during the 16 weeks of the semester, he does this exactly 112 times.

Find a good approximation for the probability that he eats 50 or more chocolate cookies during the semester.

3. Let $U_1, \ldots, U_{240}$ be 240 independent, continuous random variables, each of which are uniformly distributed on the interval $[-1, 9]$. Calculate the (approximate) probability that the sum of these 240 random variables exceeds 1000.

4. Consider a collection of random variables $X_1, \ldots, X_{80}$ that have joint probability density function $f_{X_1,\ldots,X_{80}}(x_1, \ldots, x_{80}) = 2^{80}e^{-2(x_1+\cdots+x_{80})}$ when all of the x_j's are positive, and $f_{X_1,\ldots,X_{80}}(x_1, \ldots, x_{80}) = 0$ otherwise.

4a. What kind of random variables are the X_j's?

4b. Are they independent?

4c. For each j, what are the values of the mean and variance, i.e., $\mathbb{E}(X_j)$ and $\mathrm{Var}(X_j)$?

4d. Define $Y = X_1 + \cdots + X_{80}$. What kind of random variable is Y? What are its parameters?

4e. Approximate the value of $P(Y < 45)$.