STAT/MA 41600
In-Class Problem Set #35: November 6, 2017
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Problem Set 35 Answers

1a. If $P(c < Z < c) = 0.8324$ then (by the symmetry of the distribution of Z) it follows that $P(Z \leq -c) = (1 - 0.8324)/2 = 0.0838$, so $P(Z < c) = P(-c < Z < c) + P(Z \leq -c) = 0.8324 + 0.0838 = 0.9162$. From the normal distribution table, we conclude that $c = 1.38$.
1b. Since $P(4.2 - c < Y < 4.2 + c) = 0.8324$, then $P(-\frac{c}{\sqrt{2.3}} < \frac{Y-4.2}{\sqrt{2.3}} < \frac{c}{\sqrt{2.3}}) = 0.8324$, and equivalently $P(-c/\sqrt{2.3} < Z < c/\sqrt{2.3}) = 0.8324$. Using the results from the previous problem, it follows that $c/\sqrt{2.3} = 1.38$, so $c = 2.09$.

2. We let X denote the price of gas (in dollars). We compute $P(2.50 < X < 2.75) = P(\frac{2.50-2.60}{0.10} < X < \frac{2.75-2.60}{0.10}) = P(-1 < X < 1.5) = P(X < 1.5) - P(X < -1) = P(X < 1.5) - P(X > 1) = P(X < 1.5) - (1 - P(X < 1)) = 0.9332 - (1 - 0.8413) = 0.7745$.

3. We let X denote the length of the pregnancy (in day). We compute $P(282 < X < 286) = P(\frac{282-283.6}{1.5} < X < \frac{286-283.6}{1.5}) = P(-1.07 < X < 1.6) = P(X < 1.6) - P(X < -1.07) = P(X < 1.6) - P(X > 1.07) = P(X < 1.6) - (1 - P(X < 1.07)) = 0.9452 - (1 - 0.8577) = 0.8029$.

4a. We let X denote the weight of a book (in ounces). We compute $P(X < 16) = P(X < \frac{16-14.2}{1.7}) = P(X < 1.06) = 0.8554$.
4b. We compute $P(13 < X < 15) = P(\frac{13-14.2}{1.7} < X < \frac{15-14.2}{1.7}) = P(-0.71 < X < 0.47) = P(X < 0.47) - P(X < -0.71) = P(X < 0.47) - P(X > 0.71) = P(X < 0.47) - (1 - P(X < 0.71)) = 0.6808 - (1 - 0.7611) = 0.4419$.
4c. From part 4a, the probability that a book is heavy is $1 - 0.8554 = 0.1446$. The number of heavy books is Binomial with $n = 10$ and $p = 0.1446$, so the probability of exactly 3 out of 10 books are heavy is $\binom{10}{3}(0.1446)^3(0.8554)^7 = 0.1216$.