STAT/MA 41600
In-Class Problem Set #32: October 27, 2017

1a. Suppose it is 12 noon right now, and the time until the arrival of the next taxi cab is an exponential random variable, with expected value 3 minutes. What is the probability that the next taxi cab will not arrive until 12:05 PM or later?
1b. Now suppose that no taxi cab arrived by 12:05 PM. What is the conditional probability that no taxi cab will have arrived by 12:07 PM?

2a. Consider a pair of random variables X and Y with joint probability density function $f_{X,Y}(x,y) = 21e^{-3x-7y}$ for x, y positive, and $f_{X,Y}(x,y) = 0$ otherwise. Find the probability that X is the smaller of these two random variables.
2b. Can you generalize the previous result? In other words, if λ_1, λ_2 are two fixed, positive constants, and if X and Y are a pair of random variables with joint probability density function $f_{X,Y}(x,y) = \lambda_1\lambda_2 e^{-\lambda_1 x - \lambda_2 y}$ for x, y positive, and $f_{X,Y}(x,y) = 0$ otherwise, what is the probability that X is the smaller of these two random variables?

3. Same setup as question #2a. Find the probability that X is larger than Y but smaller than $2Y$. In other words, compute $P(Y < X < 2Y)$.

4. Consider five children whose breathing patterns are independent. Suppose that, for each child, the time until the start of the child's next breath is exponential, with expected time of 3 seconds until the child's next breath.
4a. Find the probability that none of the five children start to take a breath within the next 1.2 seconds.
4b. What is the expected time until the next child in the group starts to take her/his next breath?