STAT/MA 41600
In-Class Problem Set #31: October 25, 2017

1. Consider a pair of random variables X and Y whose joint probability density function is constant on the triangle with vertices at the points $(-4, 0)$, $(0, 2)$, and $(8, 0)$. What is the probability that X is positive?

2. Suppose that

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq 5 \\ \frac{x-5}{15} & \text{if } 5 < x < 20 \\ 1 & \text{if } x \geq 20 \end{cases}$$

2a. Find $\mathbb{E}(X)$.
2b. Find $\mathrm{Var}(X)$.

3. Consider a collection of three independent random variables, U, V, W, that each have continuous uniform distribution on the interval $[-3, 3]$. Let X be the largest of these three random variables. What is the probability density function of X?

4. Suppose that X and Y are independent, continuous random variables whose probability density functions are each equal to $1/3$ on the interval $[0, 3]$, and equal to 0 otherwise.

Now define $Z = X + Y$. What is the probability density function of Z?

Hints: The random variable Z takes values in the range $[0, 6]$. It is easiest to find $f_Z(z)$ in a piecewise manner, first by handling the case $z \leq 3$ and then considering $z \geq 3$. In the latter case, it is easier to compute the complementary probability.