STAT/MA 41600
In-Class Problem Set #28: October 20, 2017
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Problem Set 28 Answers

1. We have $\mathbb{E}(X) = \int_0^3 (x)(x/9)\,dx + \int_3^6 (x)(2/3 - x/9)\,dx = x^3/27|_{x=0}^3 + (x^2/3 - x^3/27)|_{x=3}^6 = 1 + (12-8) - (3-1) = 3$.

2a. We compute $\mathbb{E}(X) = \int_0^2 \int_0^x (x)(3/4)(x-y)\,dy\,dx = \int_0^2 (x)(3/4)(xy - y^2/2)|_{y=0}^x\,dx = \int_0^2 (3/4)(x^3/2)\,dx = (3/4)(x^4/8)|_{x=0}^2 = (3/4)(2) = 3/2$.
2b. We compute $\mathbb{E}(Y) = \int_0^2 \int_0^x (y)(3/4)(x-y)\,dy\,dx = \int_0^2 (3/4)(xy^2/2 - y^3/3)|_{y=0}^x\,dx = \int_0^2 (3/4)(x^3/6)\,dx = (3/4)(x^4/24)|_{x=0}^2 = (3/4)(16/24) = 1/2$.

3. We compute that $\mathbb{E}(Y) = \int_0^\infty \int_{5y}^\infty (y)(69e^{-3x-8y})\,dx\,dy = \int_0^\infty (y)(-23e^{-3x-8y})|_{x=5y}^\infty\,dy = \int_0^\infty (y)(23e^{-23y})\,dy = (y)(-e^{-23y})|_{y=0}^\infty - \int_0^\infty (-e^{-23y})\,dy = (-e^{-23y}/23)|_{y=0}^\infty = 1/23$.

4a. We have $\mathbb{E}(X) = \int_0^2 \int_{2y-4}^{-4y+8} (x)(1/12)\,dx\,dy = \int_0^2 (x^2/2)(1/12)|_{x=2y-4}^{-4y+8}\,dy = \int_0^2 ((-4y+8)^2/2 - (2y-4)^2/2)(1/12)\,dy = \int_0^2 (6y^2 - 24y + 24)(1/12)\,dy = (2y^3 - 12y^2 + 24y)(1/12)|_{y=0}^2 = (16 - 48 + 48)(1/12) = 4/3$.
4b. We have $\mathbb{E}(Y) = \int_0^2 \int_{2y-4}^{-4y+8} (y)(1/12)\,dx\,dy = \int_0^2 (xy)(1/12)|_{x=2y-4}^{-4y+8}\,dy = \int_0^2 (y)((-4y+8) - (2y-4))(1/12)\,dy = \int_0^2 (y)(-6y+12)(1/12)\,dy = \int_0^2 (-6y^2 + 12y)(1/12)\,dy = (-2y^3 + 6y^2)(1/12)|_{y=0}^2 = (-16+24)(1/12) = 8/12 = 2/3$.