STAT/MA 41600
In-Class Problem Set #26: October 16, 2017

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = (3/4)(x-y)$ for $0 < y < x < 2$, and $f_{X,Y}(x,y) = 0$ otherwise.
1a. Verify that $f_{X,Y}(x,y)$ is a valid joint probability density function, i.e., verify that it is nonnegative and that the integral over all x's and y's is 1.
1b. Find the probability density function of X.
1c. Find the probability density function of Y.

2a. For the setup in question 1, are X and Y independent? Why or why not?
2b. For the setup in question 1, find $P(X + Y \leq 1)$. (Hint: Draw the region of integration carefully, before setting up the double-integral.)

3. Suppose that X and Y are independent, continuous random variables with $f_X(x) = 3e^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise, and with $f_Y(y) = 8e^{-8y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Find $P(X > 2Y)$.

4a. Suppose that X and Y have constant joint probability density function on the circle with center at the origin and radius 3. What is this joint probability density function?
4b. What is the probability that $X^2 + Y^2$ is larger than 4?