STAT/MA 41600
In-Class Problem Set #20/#22: October 2, 2017

1. Five couples (ten students altogether) randomly choose where to sit, in a row of ten chairs. All such arrangements of the students are equally likely. What is the expected number of couples that sit together?

2. Same situation as **1**. What is the probability that all of the couples are sitting together, i.e., that each student is seated next to her/his partner?

3. Consider a class with 40 students. Four of the students are randomly chosen to be in "Group A".
3a. How many such selections of a group of four students are available?
3b. How many such selections of a group of four students are available, if we need to include Bob in Group A?
3c. What is the probability that Bob is in Group A?

4. Suppose that 10 red bears and 10 yellow bears are placed in a bag. Ten students each draw two bears at random, without replacement. Let X denote the number of students who get two bears of the same color. Let Y denote the number of students who get two bears of different colors.
4a. Find $\mathbb{E}(X)$.
4b. Find $\mathbb{E}(Y)$.
[Note that $X + Y = 10$ always, so $10 = \mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$. Therefore, the solutions to **4a** and **4b** must sum to 10.]
4c. Find $\mathrm{Var}(X)$.