STAT/MA 41600
In-Class Problem Set #18: September 27, 2017
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Problem Set 18 Answers

1a. The probability that at least 1 biker passes in the next 15 seconds is $1-e^{-1/2}(1/2)^0/0! = 0.3935$.
1b. The probability that at least 4 bikers pass in the next 5 minutes is $1-\sum_{x=0}^{3} e^{-10}10^x/x! = 0.9897$.

2. The conditional probability is $P(X \leq 5 \mid X \geq 1) = P(1 \leq X \leq 5)/P(X \geq 1) = (\sum_{x=1}^{5} e^{-5}5^x/x!)/(1 - e^{-5}5^0/0!) = 0.6134$.

3. We compute $P(Y \geq X) = \sum_{x=0}^{\infty}\sum_{y=x}^{\infty}(e^{-2}2^x/x!)(1/2)^y - e^{-2} = \sum_{x=0}^{\infty}(e^{-2}2^x/x!)(1/2)^x/(1-1/2) - e^{-2} = 2e^{-2}\sum_{x=0}^{\infty} 1/x! - e^{-2} = 2e^{-2}e^1 - e^{-2} = 2e^{-1} - e^{-2} = 0.6004$.

Alternatively, we have $P(Y \geq X) = P(X = 0) + P(Y \geq X \geq 1) = e^{-2}2^0/0! + \sum_{x=1}^{\infty}\sum_{y=x}^{\infty}(e^{-2}2^x/x!)(1/2)^y = e^{-2} + \sum_{x=1}^{\infty}(e^{-2}2^x/x!)(1/2)^x/(1-1/2) = e^{-2} + 2e^{-2}\sum_{x=1}^{\infty} 1/x! = e^{-2} + 2e^{-2}(e^1 - 1) = 2e^{-1} - e^{-2} = 0.6004$.

4. We define $X = X_1 + \cdots + X_5$. The sum of independent Poisson random variables is a Poisson random variable. We have $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_5) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_5) = (5)(1.7) = 8.5$. Thus, X is a Poisson random variable with mean 8.5. So we get $P(X \geq 6) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5} e^{-8.5}(8.5)^x/x! = 0.8504$.