STAT/MA 41600
In-Class Problem Set #17: September 25, 2017
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Problem Set 17 Answers

1ab. The probability of having a triple on a round is $p = 1/6^2 = 1/36$. Thus, the number of rounds is Negative Binomial with $r = 10$ and $p = 1/36$. So the expected number of rounds is $r/p = 360$ and the variance is $(r)(1-p)/p^2 = 12{,}600$.
1c. Let X denote the number of rounds. Then $P(X \le 13) = P(X = 10) + P(X = 11) + P(X = 12) + P(X = 13) = \binom{9}{9}(1/36)^{10}(35/36)^0 + \binom{10}{9}(1/36)^{10}(35/36)^1 + \binom{11}{9}(1/36)^{10}(35/36)^2 + \binom{12}{9}(1/36)^{10}(35/36)^3 = 7.2448 \times 10^{-14}$.

2. Let X denote the number of days until he has eaten 5 chocolate cookies. Then $P(X > 7) = 1 - P(X \le 7) = 1 - P(X = 5) - P(X = 6) - P(X = 7) = 1 - \binom{4}{4}(0.40)^5(0.60)^0 - \binom{5}{4}(0.40)^5(0.60)^1 - \binom{6}{4}(0.40)^5(0.60)^2 = 0.9037$.

3a. We compute $P(X > 8 \mid X > 6) = \frac{P(X>8 \ \&\ X>6)}{P(X>6)} = \frac{P(X>8)}{P(X>6)} = \frac{1-P(X\le 8)}{1-P(X\le 6)}$, or equivalently, $\frac{1-P(X=8)-P(X=7)-P(X=6)-P(X=5)}{1-P(X=6)-P(X=5)} = \frac{1-\binom{7}{4}(0.40)^5(0.60)^3-\binom{6}{4}(0.40)^5(0.60)^2-\binom{5}{4}(0.40)^5(0.60)^1-\binom{4}{4}(0.40)^5(0.60)^0}{1-\binom{5}{4}(0.40)^5(0.60)^1-\binom{4}{4}(0.40)^5(0.60)^0} = 797/925 = 0.8616$.
3b. We compute $P(X \le 8 \mid X > 6) = \frac{P(X\le 8 \ \&\ X>6)}{P(X>6)}$ or equivalently, $\frac{P(X=8)+P(X=7)}{1-P(X=6)-P(X=5)} = \frac{\binom{7}{4}(0.40)^5(0.60)^3+\binom{6}{4}(0.40)^5(0.60)^2}{1-\binom{5}{4}(0.40)^5(0.60)^1-\binom{4}{4}(0.40)^5(0.60)^0} = 0.1384$.
3c. We compute $P(X = 8 \mid X > 6) = \frac{P(X=8 \ \&\ X>6)}{P(X>6)}$ or equivalently, $\frac{P(X=8)}{1-P(X=6)-P(X=5)} = \frac{\binom{7}{4}(0.40)^5(0.60)^3}{1-\binom{5}{4}(0.40)^5(0.60)^1-\binom{4}{4}(0.40)^5(0.60)^0} = 0.0807$.

4. We have:

$P(X = 0) = (1/6)(\binom{6}{0}(1/2)^6 + \binom{5}{0}(1/2)^5 + \binom{4}{0}(1/2)^4 + \binom{3}{0}(1/2)^3 + \binom{2}{0}(1/2)^2 + 1/2) = 21/128$
$P(X = 1) = (1/6)(\binom{6}{1}(1/2)^6 + \binom{5}{1}(1/2)^5 + \binom{4}{1}(1/2)^4 + \binom{3}{1}(1/2)^3 + \binom{2}{1}(1/2)^2 + 1/2) = 5/16$
$P(X = 2) = (1/6)(\binom{6}{2}(1/2)^6 + \binom{5}{2}(1/2)^5 + \binom{4}{2}(1/2)^4 + \binom{3}{2}(1/2)^3 + (1/2)^2) = 33/128$
$P(X = 3) = (1/6)(\binom{6}{3}(1/2)^6 + \binom{5}{3}(1/2)^5 + \binom{4}{3}(1/2)^4 + (1/2)^3) = 1/6$
$P(X = 4) = (1/6)(\binom{6}{4}(1/2)^6 + \binom{5}{4}(1/2)^5 + (1/2)^4) = 29/384$
$P(X = 5) = (1/6)(\binom{6}{5}(1/2)^6 + (1/2)^5) = 1/48$
$P(X = 6) = (1/6)(1/2)^6 = 1/384$