STAT/MA 41600
In-Class Problem Set #16: September 22, 2017

1. Suppose that, in each round of a game, Alejandra rolls three dice simultaneously. She continues rolling all three dice, until the first round in which a triple appears (i.e., until the first round that all three dice have the same value), and then she stops afterward.
1a. What is the expected number of rounds?
1b. What is the variance of the number of rounds?
1c. Alejandra likes these games of chance so much that she decides to open a casino. She will require a person to buy one ticket for *every individual round* that they want to try rolling the 3 dice. For a reward, she will give a $100 prize on each individual round in which a participant rolls a matching triple. How much should she charge for a ticket, to make this a fair game?

2. Starting on Monday, Carlos randomly grabs a cookie as he exits the dining court at lunch. Assume that 40% of the cookies are chocolate, and that his picks are independent from day to day.
2a. What is the probability that he does not get a chocolate cookie at all, during the first 5 days (Monday through Friday)?
2b. What is the probability that his first chocolate chip cookie arrives this Thursday or later? (Equivalently, what is the probability that he does not get a chocolate cookie on Monday, Tuesday, or Wednesday this week?)
2c. Carlos really wants a chocolate cookie. He is willing to continue selecting cookies at random (one per day) until he finally gets one. The dining court at his university is open seven days a week. What is the probability that he finally gets his first chocolate chip cookie on a Monday?

3. Returning to the cookie setup from question **2**: Suppose that Alice and Bob join Carlos in his activities of randomly eating 1 cookie per day at the dining court. Let X denote the number of days needed until the first chocolate cookie is eaten (by anybody in this group).
3a. Find $P(X = 5)$, i.e., find the probability that Alice, Bob, and Carlos do not get chocolate cookies during Monday through Thursday of the first week, and at least one of them gets a chocolate cookie on Friday.
3b. What is the expected value of X?

4. Suppose that X and Y are independent Geometric random variables, with $\mathbb{E}(X) = 1/p$ and $\mathbb{E}(Y) = 1/r$.
4a. What is $P(X > Y)$?
4b. What is $P(Y > X)$? [Hint: This is easy if you solve **4a**; just switch the p and r.]
4c. What is $P(Y = X)$? [Hint: You can double-check your work by making sure that your three answers sum up to 1.]