STAT/MA 41600

In-Class Problem Set #12: September 18, 2017

1. Roll a 4-sided die and a 6-sided die. Let X denote the sum.
1a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as in Problem Set #10, question 1.
1b. Using your knowledge of $\mathbb{E}(X^2)$ and $\mathbb{E}(X)$, calculate $\mathrm{Var}(X)$.
1c. Use X_1 and X_2 from Problem Set 11 to find $\mathrm{Var}(X)$ in a different way, namely, using the fact that $X = X_1 + X_2$, where the X_j's are **independent** random variables. First find $\mathrm{Var}(X_1)$ and $\mathrm{Var}(X_2)$, and then, since X_1 and X_2 are independent, we obtain $\mathrm{Var}(X) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$. Make sure that your solution agrees with **1b**.

2. Flip 5 fair coins. Let X denote the number of heads that appear.
2a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #10, question 2.
2b. Using your knowledge of $\mathbb{E}(X^2)$ and $\mathbb{E}(X)$, calculate $\mathrm{Var}(X)$.
2c. Use $X_1, \ldots, X_5$ from Problem Set 11 to find $\mathrm{Var}(X)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$, where the X_j's are **independent** random variables. First find $\mathrm{Var}(X_j)$, and then, since the X_j's are independent, we obtain $\mathrm{Var}(X) = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_5)$. Make sure that your solution agrees with **2b**.

3a. Suppose we draw 5 cards at random, without replacement, from a deck of 52 cards (such a deck includes 4 Queens). Let X denote the number of Queens drawn.
3a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as in Problem Set #10, part 3a.
3b. Use $X_1, \ldots, X_5$ from Problem Set 11 to find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$, where the X_j's are **dependent** random variables. Make sure that your solution agrees with **3a**.
3c. Using your knowledge of $\mathbb{E}(X^2)$ and $\mathbb{E}(X)$, calculate $\mathrm{Var}(X)$.
****

This time draw the 5 cards one at a time, with replacement (and shuffling) between cards.
3d. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as in Problem Set #10, part 3b.
3e. Using your knowledge of $\mathbb{E}(X^2)$ and $\mathbb{E}(X)$, calculate $\mathrm{Var}(X)$.
3f. Use $X_1, \ldots, X_5$ from Problem Set 11 to find $\mathrm{Var}(X)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$, where the X_j's are **independent** random variables. First find $\mathrm{Var}(X_j)$, and then, since the X_j's are independent, we obtain $\mathrm{Var}(X) = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_5)$. Make sure that your solution agrees with **3e**.

4. A family with three daughters and three sons needs to go to the grocery store. Besides the father, who is driving the car, exactly three of the children can come along to the grocery store with him. Suppose that the three children to join the father are chosen randomly, and all such choices are equally likely.

Let X denote the number of daughters who accompany the father to the grocery.
4a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as in Problem Set #8, question 4.
4b. Use X_1, X_2, X_3 from Problem Set 11 to find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2 + X_3$, where the X_j's are **dependent** random variables. Make sure that your solution agrees with **4a**.
4c. Using your knowledge of $\mathbb{E}(X^2)$ and $\mathbb{E}(X)$, calculate $\mathrm{Var}(X)$.