STAT/MA 41600
In-Class Problem Set #44: December 7, 2016

1. Let U be a continuous random variable that is uniformly distributed on the interval $(0, 3)$. Let $X = -\ln(U) + \ln(3) = -\ln(U/3)$.
1a. Find $P(X > 5)$.
1b. If $a > 0$, find $P(X > a)$.
1c. What kind of distribution does X have? What is/are the parameter(s)?

2. Let X denote the waiting time, in minutes, until the next message arrives on Facebook. Suppose that X is an exponential random variable with $\mathbb{E}(X) = 0.25$. Now define $Y = X^2$.
2a. What is the probability density function of Y?
2b. What is $\mathbb{E}(Y)$? (You can use u-sub, with $u = \sqrt{y}$, and then integration by parts twice.)
2c. Use what you know about $\mathbb{E}(X)$ and $\mathrm{Var}(X)$ to find $\mathbb{E}(X^2)$. Does it agree with $\mathbb{E}(Y)$?

3. If U is a continuous random variable that is uniformly distributed on the interval $(0, 2)$, define $X := U^3$.
3a. Find the probability density function of X.
3b. Use your solution to 3a to find $\mathbb{E}(X)$.
3c. Use the density of U to find $\mathbb{E}(U^3)$ directly. Make sure your answers to 3b and 3c agree.

4. Suppose that the joint distribution of X and Y is uniform (i.e., the joint density is constant) in the quadrilateral shaped region of the (x, y)-plane with vertices at $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 4)$.
4a. Find $\mathbb{E}(X)$.
4b. Find $\mathbb{E}(Y)$.
4c. Find $\mathbb{E}(XY)$.
4d. Use your solutions to parts 4a, 4b, 4c to find the covariance of X and Y.